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# Chi-Square Test in Time Series Data

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***Authors' contributions***

*This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.*

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## Abstract

This study examines the application of Chi-Square test in time series data which considers the mixed model structure and linear trending curve. The Chi-Square test is applied to the seasonal variances of the Buys-Ballot table. The emphasis is to assess the validity of the Chi-Square test using empirical examples. One hundred simulated numerical examples are used to illustrate the applicability of the Chi-Square test. Using empirical example, the Chi-Square test successfully recorded 100% of times for the mixed model. This expresses a high degree of confidence in the Chi-Square test.

*Keywords: Buys-Ballot method; descriptive time series; mixed model; linear trend, seasonal variance; choice of model.*

## 1 Introduction

The purpose of descriptive time series analysis is to isolate the four time series components available in the series. That is to de-compose an observed series ( $X_t, t = 1,\ 2,\ \ldots, n$ ) into components consisting the trend $(T_t)$, the seasonal ( $S_t$ ), the cyclical $(C_t)$ and irregular $(e_t)$ (Kendal and Ord [1], Chatfield [2].

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The decomposition models are the additive, multiplicative and mixed models. For short series, the cyclical component is superimposed into trend (Chatfield [2]) and observed series ( $X_t, t = 1, 2, ..., n$ ) can be decomposed into the trend-cycle component ( $M_t$ ), seasonal component ( $S_t$ ) and irregular component $(e_t)$. Hence, the decomposition models are

Additive Model $X_t = M_t + S_t + e_t$ (1)

Multiplicative Model $X_t = M_t \times S_t \times e_t$ (2)

Mixed Model $X_t = M_t \times S_t + e_t$ (3)

It is assumed that the seasonal indices, when exist, has period s, that is it repeats after s time periods.

$$S_{t+s} = S_{t,} \ \ for \ all \ t \tag{4}$$

For equation (1), the assumption is that, the sum of seasonal indices over a complete period is zero, ie.

$$\sum_{j=1}^{s} S_{t+j} = 0 \tag{5}$$

Again, for equations (2) and (3), the assumption is that, the sum of seasonal indices over a complete period is s.

$$\sum_{j=1}^{s} S_{t+1} = s \tag{6}$$

The assumption is that, the irregular component $e_t$ is the Gaussian $N\left(0, \sigma_1^2\right)$ white noise for equations (1) and (3), while for equation (2), $e_t$ *is the Gaussian* $N\left(1, \sigma_2^2\right)$ white noise and that $Cov\left(e_t, e_{t+k}\right) = 0, \ \forall k \neq 0$

One greatest problems identified in the use descriptive time series analysis is choice of suitable model for decomposition of any study series. That is when to use any of the three time series model is uncertain. It is important to note that, use of wrong model will definitely lead to erroneous estimate of the components.

To select among additive, multiplicative and mixed models, many scholars have suggested different approaches. Puerto and Rivera [3] proposed the use of coefficient of variation of seasonal differences CV(d) and seasonal quotient CV(c) for choice of model. According to them, additive model is appropriate, if CV(c) is greater than CV(d) and it is multiplicative if CV(c) is less than or equal to CV(d). However, they did not provide any statistical test to justify the use. Chatfield [2] suggested the use of time plot to choose between additive and multiplicative models. However, no theoretical basis was proposed for the decision rule. Iwueze, et al. [4] proposed the use of the relationship between the seasonal average ( $\bar{X}_{.j}, j = 1, 2, ...s$ ) and the seasonal standard deviations ( $\hat{\sigma}_{.j}, j = 1, 2, ...s$ ) to choose the appropriate model for decomposition.

In proposing the Chi-Square test Nwogu, et al. [5] and Dozie, et al [6] assumed that (i) the underlying distribution of the variable, $X_{ij}, \ i = 1, 2, ...m, \quad j = 1, 2, ...s,$ under study is normal. (ii) the observations in each column, $X_{ij}, \ i = 1, 2, ...m$ are independent and (iii) that the s-column are independent. Therefore, they proposed use of Chi-Square test as a basis for choosing between mixed and multiplicative models. The seasonal variance for the mixed model is $\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ . Therefore, the test for choice between mixed and multiplicative model may be reduced to identify the mixed model whose seasonal variance is simply constant multiply of the square of seasonal effect.

The aim of this article is to assess the validity of the Chi-Square test using empirical examples. The rationale of the article is that it will help to improve existing methodology by providing analyst with objective Chi-Square test in a series when it exists. The article is limited to series when trend cycle component is linear.

# 2 Methodology

Chi-Square test is applied to the seasonal variances of the Buys-Ballot table and the model structure is mixed. Hence, the summary of the seasonal variance of the Buys-Ballot table for the mixed model derived by Nwogu, et al. [5] and Dozie, et al. [6] is shown in Table 1.

**Table 1.**

| **Linear trending curve** | **Seasonal Variance** $(\sigma_{.j}^2)$ |
|---|---|
| $a+bt$ | $\frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$ |

In this arrangement, ***n*** is the total number of observations, ***s*** is the seasonal lag (number of columns), ***b*** is the slope, $s_j$ is the seasonal indices, $\sigma_1^2$ is the error variance, assumed equal to 1. Both the *b* and $s_j$ are derivable from the seasonal average.

$$\bar{X}_{.j} = \left[ a + b\left(\frac{n-s}{2}\right) + b_j \right] * S_j \tag{7}$$

$$\equiv \left[ \alpha + \beta_j \right] \times S_j \tag{8}$$

where,

$$\alpha = a + b\left(\frac{n-s}{2}\right), \beta = b \tag{9}$$

Estimates of $\alpha$ *and* $\beta$ are derivable from the regression of $\bar{X}_{.j}$ *on* $j$ *and estimates of* $S_j$ is

$$\hat{S}_j = \frac{\bar{X}_{.j}}{\hat{\alpha} + \hat{\beta}_j} \tag{10}$$

For details of Buys-Ballot procedure, see Iwueze et.al [4], Nwogu et al. [5], Dozie et al. [6], Akpanta and Iwueze [7], Dozie and Ijeomah [8], Dozie and Nwanya [9], Dozie [10], Dozie and Uwaezuoke[11], Dozie and Ibebuogu [12], Dozie and Ihekuna [13] .

## 2.1 Chi-Square Test

The expression of the seasonal variance for the mixed model is $\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2$. Therefore, Chi-Square test proposed by Nwogu, et al. [5] and Dozie, et al. [6] when trend-cycle component is linear reduces to that of test null hypothsis.

$$H_0: \ \sigma_j^2 = \sigma_{zj}^2$$

and the suitable model is mixed

$$H_1: \ \sigma_j^2 \neq \sigma_{zj}^2$$

and the suitable model is not mixed

$\sigma_j^2 = (j = 1, \ 2, ...s)$ *is the true variance of the jth season.*

$$\sigma_{zj}^2 = \frac{b^2 n(n+s)}{12} S_j^2 + \sigma_1^2 \tag{11}$$

and $\sigma_1^2$ *is the error variance assumed to be equal to* $1$

Therefore, the statistic is $\chi_c^2 = \frac{(m-1)\sigma_j^2}{\sigma_{zj}^2}$ (12)

follows the chi-square distribution with $m-1$ degree of freedom, m is the number of observations in each column and s is the seasonal lag.

The interval (12) with $100(1-\alpha)$% degree of confidence.

## 2.2 Levene's Test for Constant Variance

The Levene's test statistic for the null hypothesis

$$H_0 : \sigma_i^2 = \sigma_j^2$$

$H_1 : \sigma_i^2 \neq \sigma_j^2$ *for at least one* $i \neq j$ is defined as

$$W = \frac{(N-K)\sum_{i=1}^{k} N_i \left( \bar{z}_i - \bar{z}_{..} \right)^2}{(k-1)\sum_{i=1}^{k}\sum_{j=1}^{N_i} \left( z_{ij} - \bar{z}_i \right)^2} \tag{13}$$

where k is the number of different groups, $N_i$ *is the number of cases* in the *ith group*, $Y_{ij}$ *is the value of the jth* observation in the *ith group*.

$z_{ij}$ may be defined as deviation of $y_{ij}$ from the mean $(\bar{y}_i)$ or from the median $(\bar{y}_i)$. That is

$$z_{ij} = y_{ij} - \bar{y}_i \;\; or \;\; y_{ij} - \bar{y}_i \tag{14}$$

$$\bar{z}_{i.} = \frac{1}{N_i}\sum_{j=1}^{N_i} z_{ij} \text{ is the mean of the } z_{ij} \text{ for group } i \tag{15}$$

$$\bar{z}_{..} = \frac{1}{N}\sum_{i=1}^{k}\sum_{j=1}^{N_i} z_{ij} \text{ is mean of all } z_{ij}. \tag{16}$$

The test statistic W approximately follows the F-distribution with $k-1$ *and* $N-K$ degree of freedom. To suit the Buys-Ballot procedure, the levene's test statistic is modified with

$N = ms,\; k = s\;\; N_i = m$ as

$$W = \frac{(ms-s)}{s-1}\left[\frac{\sum_{j=1}^{s} m(\bar{z}_i - \bar{z}_{..})^2}{\sum_{j=1}^{s}\sum_{i=1}^{m}\left(z_{ij} - \bar{z}_{.j}\right)^2}\right] \tag{17}$$

$$= \frac{s(m-1)}{s-1}\left[\frac{m\sum_{i=1}^{s}\left(\bar{z}_i - \bar{z}_{..}\right)^2}{\sum_{i=1}^{s}\sum_{j=1}^{m}\left(z_{ij} - \bar{z}_{.j}\right)^2}\right] \tag{18}$$

# 3 Empirical Examples

This section discusses some of the empirical examples to illustrate the validity of the Chi-Square test. The empirical examples consist of both simulated and real life data. Sections 3.1 and 3.2 contain simulations results for the mixed model and real life example respectively.

## 3.1 Simulations Results using the Mixed Model

The simulated series used consists 100 data set of 120 observations each simulated from

$$X_t = (a+bt)\times S_t + e_t,\;\; with\; a=1,\; b=0.02,\quad e_t \sim N(0,1).\; with\; S_1 = 0.98,\; S_2 = 0.80,\; S_3 = 0.88,$$

$$S_4 = 1.04,\; S_5 = 0.96,\; S_6 = 1.22,\; S_7 = 1.27,\; S_8 = 1.32\;\; S_9 = 0.96\;\; S_{10} = 0.80,\; S_{11} = 0.88,\; S_{12} = 0.89$$

Using the MINITAB 17.0 version. The results of calculated value of the statistic from the simulated series are given in Table 2. The critical values at 5% level of significance and which for m-1 =9 degree of freedom, equal to 2.7 and 19.0. Under the null hypothesis that the appropriate model is mixed, the calculated value of the statistic in (12) is expected to lie within the interval, otherwise, it will be concluded that the data does not admit mixed model. The calculated values of the statistic from the simulated series are listed in Table 2. When compared with the interval 2.7 and 19.0, the calculated values of the statistic lie within the interval in 100 out of the 100 stimulations. This shows that the test is capable of identifying the mixed model successfully 100 percent of the times. This expresses a high degree of confidence in the Chi-Square test.

**Table 2. Calculated Chi-Square for Mixed Model: The critical values for m -1 = 9 degree of freedom are 2.7 and 19**

| Col | Series | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** | **9** | **10** |
| 1 | 10.4821 | 5.0921 | 7.5963 | 7.7977 | 7.1774 | 9.3257 | 9.4257 | 3.1520 | 3.7203 | 9.0872 |
| 2 | 9.9157 | 10.8037 | 14.0504 | 3.8469 | 6.9436 | 5.4959 | 5.4413 | 12.9076 | 15.0742 | 12.7306 |
| 3 | 13.0416 | 5.1566 | 11.2798 | 8.6702 | 13.7272 | 8.7078 | 13.9401 | 6.8329 | 8.4794 | 6.9740 |
| 4 | 6.9085 | 12.2505 | 12.4492 | 14.8293 | 4.9423 | 7.2650 | 7.9545 | 7.3044 | 12.3286 | 12.3963 |
| 5 | 16.1748 | 10.4191 | 8.4801 | 10.5545 | 4.2798 | 17.1571 | 3.9333 | 5.2740 | 4.9656 | 12.9938 |
| 6 | 8.2899 | 16.4968 | 6.5897 | 3.3076 | 7.5989 | 9.7034 | 7.8557 | 15.1125 | 7.5076 | 6.6820 |
| 7 | 8.0266 | 10.3324 | 9.5820 | 16.5750 | 16.5760 | 4.9320 | 6.0845 | 9.1149 | 6.6597 | 12.0505 |
| 8 | 16.9952 | 2.5788 | 15.9770 | 9.7204 | 7.0275 | 13.3809 | 14.6644 | 9.7687 | 17.1803 | 9.2279 |
| 9 | 7.5575 | 3.3121 | 4.8592 | 7.8997 | 3.3922 | 4.2077 | 8.2510 | 4.6430 | 8.8565 | 8.9871 |
| 10 | 7.0924 | 14.5750 | 5.2015 | 8.1676 | 15.4410 | 11.3758 | 8.8120 | 13.1021 | 6.3318 | 3.9990 |
| 11 | 15.7355 | 12.9050 | 12.1345 | 6.8173 | 12.7856 | 4.7580 | 6.8402 | 6.0106 | 5.0302 | 7.8786 |
| 12 | 4.8516 | 8.6963 | 10.7143 | 2.7000 | 18.1478 | 5.0776 | 3.2408 | 12.7302 | 6.2707 | 8.8515 |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept |

**Table 2. Calculated chi-square for mixed model**

| Col | Series | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **11** | **12** | **13** | **14** | **15** | **16** | **17** | **18** | **19** | **20** |
| 1 | 9.6094 | 14.2685 | 8.9455 | 4.3441 | 7.5303 | 13.2527 | 10.9166 | 5.9109 | 11.4996 | 4.7858 |
| 2 | 11.2284 | 10.0976 | 5.4781 | 12.1527 | 6.7144 | 8.8954 | 7.5931 | 7.3266 | 10.3937 | 6.3749 |
| 3 | 7.0312 | 5.5312 | 6.2797 | 15.7758 | 9.4296 | 5.5272 | 4.8595 | 10.4684 | 5.0037 | 2.0421 |
| 4 | 5.9895 | 9.5869 | 10.2811 | 9.0821 | 12.0272 | 8.1651 | 5.9885 | 11.6237 | 8.5224 | 7.9510 |
| 5 | 6.5379 | 4.7852 | 6.8220 | 7.7652 | 3.9335 | 6.2330 | 8.0709 | 4.5609 | 7.6014 | 13.8613 |
| 6 | 13.8824 | 5.7359 | 6.9559 | 6.2243 | 13.9755 | 9.0818 | 7.1403 | 13.1169 | 4.8691 | 5.9653 |
| 7 | 6.1482 | 7.9960 | 14.7312 | 13.7218 | 8.2492 | 11.2175 | 10.9352 | 16.8010 | 6.7297 | 8.7209 |
| 8 | 15.7446 | 5.4835 | 7.6453 | 6.5535 | 12.3519 | 5.6445 | 8.2929 | 5.1867 | 4.7716 | 17.7810 |
| 9 | 6.3454 | 6.6425 | 6.7641 | 13.7449 | 5.9355 | 8.0525 | 8.9194 | 4.1364 | 10.5184 | 7.6881 |
| 10 | 9.0031 | 7.5444 | 12.1063 | 5.8258 | 8.6373 | 11.5941 | 9.8175 | 15.5784 | 5.3195 | 4.6014 |
| 11 | 4.7313 | 16.6570 | 7.5044 | 3.1541 | 15.2454 | 6.3591 | 4.1399 | 4.2341 | 7.3711 | 10.6682 |
| 12 | 9.2021 | 7.9693 | 15.0332 | 9.3116 | 5.2108 | 6.6369 | 10.7285 | 10.6694 | 18.1081 | 7.5191 |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept |

**Table 2. Calculated chi-square for mixed model**

| Col | Series | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| 1 | 4.9925 | 6.6872 | 9.3019 | 8.4440 | 16.1231 | 4.6607 | 5.6206 | 12.8225 | 7.5324 | 7.2493 |
| 2 | 9.9520 | 7.6571 | 5.1852 | 3.8616 | 5.1936 | 16.0880 | 6.3393 | 9.3057 | 7.9772 | 11.2456 |
| 3 | 15.7739 | 9.2675 | 7.5613 | 7.3520 | 9.5426 | 3.5049 | 7.1022 | 7.4936 | 6.2753 | 13.7839 |
| 4 | 7.9491 | 8.8898 | 6.4351 | 3.4607 | 4.3826 | 8.9271 | 5.1558 | 6.6446 | 5.9960 | 4.3721 |
| 5 | 15.4095 | 7.8317 | 12.7285 | 2.6768 | 5.3428 | 8.9074 | 8.3408 | 14.2036 | 6.3303 | 5.2609 |
| 6 | 4.4069 | 11.7922 | 7.8317 | 16.1478 | 8.3204 | 10.8279 | 17.3949 | 7.7486 | 8.6958 | 6.9722 |

| **Col** | **Series** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **21** | **22** | **23** | **24** | **25** | **26** | **27** | **28** | **29** | **30** |
| 7 | 10.5531 | 10.0569 | 9.3096 | 14.4791 | 7.5278 | 4.7670 | 7.9232 | 10.5640 | 13.9507 | 8.6635 |
| 8 | 9.1184 | 5.5631 | 12.5454 | 7.8327 | 11.9395 | 9.5254 | 8.5931 | 9.3800 | 10.2167 | 8.2705 |
| 9 | 8.0141 | 8.2588 | 7.5039 | 6.1646 | 9.3666 | 5.1337 | 5.9292 | 8.7776 | 6.7719 | 7.2804 |
| 10 | 7.0412 | 3.1853 | 8.2615 | 11.9400 | 7.4869 | 5.5111 | 7.5531 | 13.3698 | 15.8669 | 10.7257 |
| 11 | 2.3845 | 8.1368 | 4.6635 | 3.9207 | 6.7324 | 14.2902 | 6.2990 | 8.0857 | 17.5143 | 10.1542 |
| 12 | 8.7773 | 15.5007 | 9.8291 | 3.6141 | 10.8442 | 10.7897 | 12.2908 | 10.2846 | 13.0388 | 10.5793 |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept |

**Table 2. Calculated chi-square for mixed model**

| **Col** | **Series** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **31** | **32** | **33** | **34** | **35** | **36** | **37** | **38** | **39** | **40** |
| 1 | 4.3301 | 5.2052 | 11.6108 | 8.4196 | 13.2948 | 8.7634 | 9.7346 | 7.9785 | 7.0880 | 5.2840 |
| 2 | 12.5402 | 8.0585 | 4.8702 | 8.7460 | 3.7832 | 3.6429 | 6.8156 | 7.5950 | 2.7919 | 18.6303 |
| 3 | 5.3371 | 6.7496 | 7.6270 | 5.8716 | 6.0977 | 4.2432 | 3.7521 | 3.4111 | 8.3039 | 3.9887 |
| 4 | 9.4842 | 14.6409 | 2.7816 | 6.4014 | 6.2389 | 2.0595 | 5.8052 | 5.1196 | 6.9820 | 3.5517 |
| 5 | 3.4534 | 8.2087 | 11.4106 | 4.0613 | 5.8798 | 6.6919 | 8.6991 | 2.1725 | 12.3841 | 10.2771 |
| 6 | 19.0150 | 8.7136 | 5.6496 | 14.7791 | 2.0007 | 17.3150 | 10.3676 | 17.1883 | 8.5542 | 5.1544 |
| 7 | 13.7678 | 17.1864 | 9.5216 | 11.3132 | 5.1484 | 7.9064 | 13.3484 | 6.0798 | 0.6810 | 4.6176 |
| 8 | 11.9285 | 9.2793 | 9.5300 | 10.9811 | 6.4705 | 8.3198 | 0.1218 | 4.7634 | 8.9523 | 7.3645 |
| 9 | 12.2726 | 7.0765 | 12.9150 | 11.2232 | 9.3524 | 6.5209 | 5.8827 | 10.9141 | 4.8175 | 14.1636 |
| 10 | 4.3265 | 9.1949 | 6.7857 | 10.5474 | 0.8313 | 17.9135 | 2.8196 | 8.8829 | 7.6546 | 6.6831 |
| 11 | 10.4583 | 3.4714 | 9.7626 | 5.9101 | 0.5449 | 6.0276 | 2.9715 | 12.7119 | 7.4249 | 4.9223 |
| 12 | 8.4716 | 4.9938 | 9.8488 | 5.7529 | 8.1471 | 4.4695 | 5.8214 | 3.4978 | 3.6641 | 7.1728 |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept |

**Table 2. Calculated chi-square for mixed model**

| **Col** | **Series** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **41** | **42** | **43** | **44** | **45** | **46** | **47** | **48** | **49** | **50** |
| 1 | 10.3691 | 8.5130 | 7.0330 | 4.3074 | 7.3176 | 6.8423 | 6.7538 | 9.7079 | 6.4727 | 7.0871 |
| 2 | 11.6339 | 7.8723 | 6.2565 | 12.1941 | 1.3870 | 3.4437 | 6.2822 | 0.9314 | 5.9619 | 7.611 |
| 3 | 4.4322 | 9.6867 | 14.3807 | 3.1714 | 9.9120 | 3.7780 | 0.9761 | 7.3828 | 0.6245 | 8.2764 |
| 4 | 8.5585 | 4.6870 | 4.5882 | 9.7055 | 2.9632 | 8.8073 | 4.5320 | 7.9623 | 5.9960 | 7.0251 |
| 5 | 5.6940 | 4.8856 | 10.7079 | 6.3863 | 0.2996 | 4.3244 | 5.4493 | 0.7636 | 12.7787 | 5.6149 |
| 6 | 7.5931 | 11.1882 | 5.7294 | 4.9469 | 9.5238 | 6.4484 | 9.4625 | 6.7796 | 5.4770 | 7.0564 |
| 7 | 15.1459 | 3.8121 | 0.5215 | 14.1749 | 1.2227 | 12.5596 | 8.8130 | 6.9940 | 8.6699 | 7.7016 |
| 8 | 6.3324 | 8.1706 | 2.8706 | 14.4109 | 4.4109 | 14.4099 | 7.0864 | 4.2041 | 1.1815 | 8.7890 |
| 9 | 7.8135 | 3.4254 | 9.6682 | 9.4238 | 8.8953 | 7.4470 | 6.5467 | 5.0319 | 5.9216 | 11.6033 |
| 10 | 3.0301 | 10.6712 | 6.3942 | 15.3995 | 7.6932 | 10.7740 | 4.9069 | 14.8724 | 3.6535 | 18.5999 |
| 11 | 11.0178 | 18.1801 | 8.6928 | 11.8529 | 16.1330 | 12.1989 | 12.4033 | 4.3272 | 5.8165 | 5.5348 |
| 12 | 12.5979 | 10.8027 | 7.3551 | 12.0797 | 6.6074 | 7.7126 | 8.1004 | 5.4474 | 8.1823 | 10.3745 |
| Decision | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept | Accept |

## 3.2 Real life example

The real life example is based on the monthly time series data on church marriages in Owerri, Imo State, Nigeria, for the period 2008 to 2019 given in Appendix A while the time plots are in Figs 1 and 2. The column (monthly) variances are shown in Table 3. The first step is to check whether the data admits the additive model. The modified Levene's test statistic shown in (18) is used. The null hypothesis that the data admits additive model is reject if W is greater than the tabulated value, for which $\alpha = 0.05$ level of significance and $m-1=11$ degree of freedom equal to 1.82 or do not reject $H_0$ otherwise. When compared with the critical value (1.82). W is greater, suggesting that the data does not admit the additive model.

**Table 3. Deviations of the observed values from means (** $Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$ **)**

| | **Jan.** | **Feb.** | **Mar.** | **Apr.** | **May** | **Jun.** | **Jul.** | **Aug.** | **Sept.** | **Oct.** | **Nov.** | **Dec.** | **total** | $\bar{z}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2008 | 6.42 | 4.75 | 8.17 | 13.08 | 2.17 | 3.75 | 3.67 | 3.58 | 0.50 | 10.67 | 5.25 | 12.67 | 74.67 | 6.22 | 4.09 |
| 2009 | 6.42 | 1.75 | 1.83 | 2.08 | 5.17 | 0.75 | 0.67 | 3.58 | 0.50 | 2.33 | 6.75 | 6.67 | 38.50 | 3.21 | 2.42 |
| 2010 | 5.58 | 1.75 | 0.83 | 2.92 | 0.83 | 0.75 | 2.33 | 6.42 | 4.50 | 4.33 | 1.75 | 4.67 | 36.67 | 3.06 | 1.98 |
| 2011 | 3.42 | 0.25 | 4.83 | 8.08 | 3.83 | 1.25 | 1.33 | 4.42 | 3.50 | 5.33 | 2.25 | 2.33 | 40.83 | 3.40 | 2.14 |
| 2012 | 5.42 | 1.25 | 0.17 | 12.92 | 0.83 | 5.25 | 2.67 | 1.58 | 25.50 | 9.33 | 9.25 | 20.33 | 94.50 | 7.88 | 8.13 |
| 2013 | 2.58 | 0.75 | 1.17 | 16.92 | 5.17 | 6.25 | 0.33 | 12.58 | 5.50 | 16.67 | 16.25 | 3.33 | 87.50 | 7.29 | 6.50 |
| 2014 | 3.58 | 5.25 | 0.83 | 2.92 | 10.17 | 2.25 | 0.33 | 0.42 | 0.50 | 5.33 | 11.25 | 8.33 | 51.17 | 4.26 | 3.87 |
| 2015 | 7.58 | 5.25 | 1.17 | 4.92 | 3.17 | 2.75 | 0.33 | 1.58 | 2.50 | 1.33 | 5.75 | 5.33 | 41.67 | 3.47 | 2.25 |
| 2016 | 1.58 | 1.25 | 3.17 | 10.08 | 1.83 | 3.25 | 0.67 | 0.58 | 3.50 | 10.67 | 0.25 | 7.67 | 44.50 | 3.71 | 3.70 |
| 2017 | 1.58 | 2.25 | 0.17 | 3.08 | 7.83 | 4.75 | 3.33 | 2.42 | 5.50 | 13.33 | 10.75 | 0.33 | 55.33 | 4.61 | 4.12 |
| 2018 | 3.58 | 4.75 | 0.83 | 2.92 | 5.83 | 2.75 | 1.67 | 5.42 | 4.50 | 9.33 | 9.75 | 2.33 | 53.67 | 4.47 | 2.80 |
| 2019 | 4.42 | 1.75 | 4.83 | 7.08 | 4.83 | 2.75 | 1.33 | 4.42 | 4.50 | 12.67 | 9.75 | 10.67 | 69.00 | 5.75 | 3.58 |
| Total | **52.17** | **31.00** | **28.00** | **87.00** | **51.67** | **36.50** | **18.67** | **47.00** | **61.00** | **101.33** | **89.00** | **84.67** | 688.00 | | |
| $\bar{z}_{.j}$ | **4.35** | **2.58** | **2.33** | **7.25** | **4.31** | **3.04** | **1.56** | **3.92** | **5.08** | **8.44** | **7.42** | **7.06** | | 4.78 | |
| $\sigma_{.j}$ | **1.96** | **1.86** | **2.46** | **4.99** | **2.82** | **1.74** | **1.19** | **3.31** | **6.69** | **4.72** | **4.63** | **5.52** | | | 4.32 |

**Table 4. Square of deviations of the observed values from seasonal means**

$$\left( z_{ij} - \bar{z}_{.j} \right)^2$$

| | **Jan.** | **Feb.** | **Mar.** | **Apr.** | **May** | **Jun.** | **Jul.** | **Aug.** | **Sept.** | **Oct.** | **Nov.** | **Dec.** | **total** |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2008 | 4.28 | 4.69 | 34.03 | 34.03 | 4.57 | 0.50 | 4.46 | 0.11 | 21.01 | 4.94 | 4.69 | 31.48 | 148.80 |
| 2009 | 4.28 | 0.69 | 0.25 | 26.69 | 0.74 | 5.25 | 0.79 | 0.11 | 21.01 | 37.35 | 0.44 | 0.15 | 97.76 |
| 2010 | 1.53 | 0.69 | 2.25 | 18.78 | 12.06 | 5.25 | 0.60 | 6.25 | 0.34 | 16.90 | 32.11 | 5.71 | 102.47 |
| 2011 | 0.87 | 5.44 | 6.25 | 0.69 | 0.22 | 3.21 | 0.05 | 0.25 | 2.51 | 9.68 | 26.69 | 22.30 | 78.17 |
| 2012 | 1.14 | 1.78 | 4.69 | 32.11 | 12.06 | 4.88 | 1.23 | 5.44 | 416.84 | 0.79 | 3.36 | 176.30 | 660.63 |
| 2013 | 3.11 | 3.36 | 1.36 | 93.44 | 0.74 | 10.29 | 1.49 | 75.11 | 0.17 | 67.60 | 78.03 | 13.85 | 348.58 |
| 2014 | 0.58 | 7.11 | 2.25 | 18.78 | 34.35 | 0.63 | 1.49 | 12.25 | 21.01 | 9.68 | 14.69 | 1.63 | 124.46 |
| 2015 | 10.47 | 7.11 | 1.36 | 5.44 | 1.30 | 0.09 | 1.49 | 5.44 | 6.67 | 50.57 | 2.78 | 2.97 | 95.69 |
| 2016 | 7.64 | 1.78 | 0.69 | 8.03 | 6.11 | 0.04 | 0.79 | 11.11 | 2.51 | 4.94 | 51.36 | 0.37 | 95.38 |
| 2017 | 7.64 | 0.11 | 4.69 | 17.36 | 12.45 | 2.92 | 3.16 | 2.25 | 0.17 | 23.90 | 11.11 | 45.19 | 130.95 |
| 2018 | 0.58 | 4.69 | 2.25 | 18.78 | 2.33 | 0.09 | 0.01 | 2.25 | 0.34 | 0.79 | 5.44 | 22.30 | 59.86 |
| 2019 | 0.005 | 0.69 | 6.25 | 0.03 | 0.28 | 0.09 | 0.05 | 0.25 | 0.34 | 17.83 | 5.44 | 13.04 | 44.29 |
| Total | 42.14 | 38.17 | 66.33 | 274.17 | 87.21 | 33.23 | 15.63 | 120.83 | 492.92 | 244.96 | 236.17 | 335.30 | **1987.05** |

**Table 5. Calculation of $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ m = 12**

| $\bar{z}_{.j}$ | $\bar{z}_{..}$ | $\bar{z}_{.j}-\bar{z}_{..}$ | $\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ | $12\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ |
|---|---|---|---|---|
| **4.35** | 4.78 | -0.43 | 0.18 | 2.22 |
| **2.58** | 4.78 | -2.20 | 4.84 | 58.08 |
| **2.33** | 4.78 | -2.45 | 6.00 | 72.03 |
| **7.25** | 4.78 | 2.47 | 6.10 | 73.21 |
| **4.31** | 4.78 | -0.47 | 0.22 | 2.65 |
| **3.04** | 4.78 | -1.74 | 3.03 | 36.33 |
| **1.56** | 4.78 | -3.22 | 10.37 | 124.42 |
| **3.92** | 4.78 | -0.86 | 0.74 | 8.88 |
| **5.08** | 4.78 | 0.30 | 0.09 | 1.08 |
| **8.44** | 4.78 | 3.66 | 13.40 | 160.75 |
| **7.42** | 4.78 | 2.64 | 6.97 | 83.64 |
| **7.06** | 4.78 | 2.28 | 5.20 | 62.38 |
| | | | | 685.66 |

From Appendix A and Table 5

$$W=\frac{12*(12-1)(685.66)}{(12-1)(1987.05)} = \frac{90507.12}{21857.55} = 4.14$$

Having admitted that the data does not additive model, the choice now choose lies between mixed and multiplicative models. According to the chi-square test by Nwogu, et al. [5] and Dozie et al. [6] the null hypothesis that the data admits the mixed model is rejected, if the statistic defined in (12) lies outside the interval $\left[\chi^2_{\frac{\alpha}{2},(m-1)},\chi^2_{1-\frac{\alpha}{2},(m-1)}\right]$ which for $\alpha=0.05$ level of significance and $m-1=11$ degrees of freedom, equals (3.8, 21.9) or do not reject $H_0$ otherwise, and from (12) the calculated values, $\chi^2_{cal}$ given in Table 6 were obtained. When compared with the critical values (3.8 and 21.9), all the calculated values lie outside the interval, indicating that the data does not admit mixed model.

However, there is indication the choice of model may be affected by violation of the underlying assumptions, therefore, there is need to evaluate data for transformation to meet the constant variance and normality assumptions in the distribution. When the column variances of the transformed series given in Table 7 are subjected to test for constant variance, the calculated Levene's test statistic (0.78) is less than the tabulated (1.82) at $\alpha=0.05$ level of significance and $m-1=11$ degrees of freedom. This indicates that the variance is constant and the transformed series admits additive model.

**Table 6. Seasonal effects ($S_j$), estimate of the column variance ($\hat{\sigma}^2_j$) and Calculated Chi-square $\left(\chi^2_{cal}\right)$**

| j | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $S_j$ | 1.68 | 0.77 | 0.77 | 2.18 | 1.15 | 0.54 | 0.30 | 0.67 | 0.63 | 0.98 | 0.98 | 1.63 |
| $\hat{\sigma}^2_j$ | 24.5 | 10.8 | 11.9 | 82.3 | 28.2 | 13.1 | 4.1 | 27.7 | 73.0 | 100 | 81.5 | 84.8 |
| $\chi^2_{cal}$ | 0.03 | 0.06 | 0.06 | 0.05 | 0.07 | 0.14 | 0.14 | 0.19 | 0.58 | 0.32 | 026 | 0.10 |

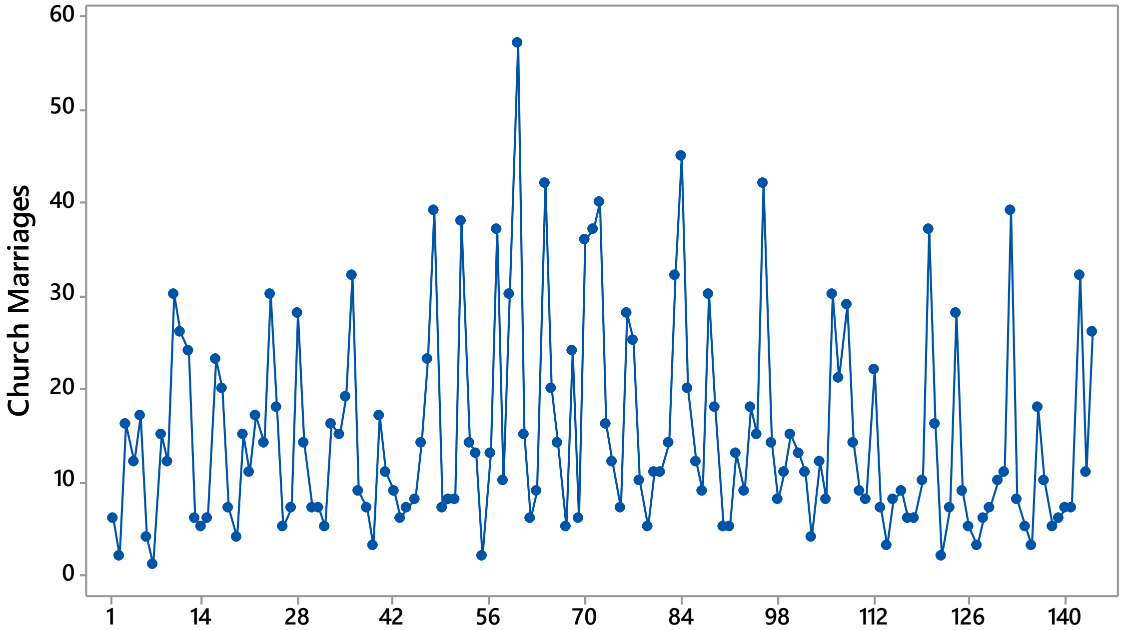


**Fig. 1. Time plot of original series of church marriages**

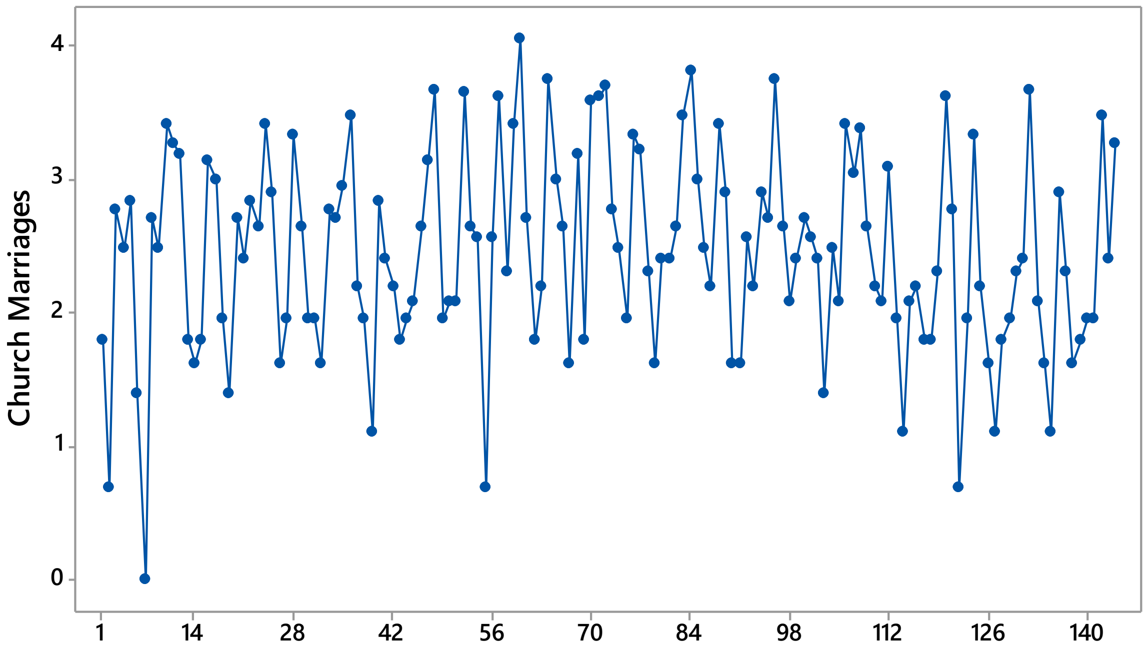


**Fig. 2. Time plot of transformed series of church marriages**

From appendix B and Table 6, $\sigma_1^2 = 1,\ b = 0.1143,\ n = 144,\ m = 12$

Therefore, from (6), $\sigma_{zj}^2 = (0.1143)^2 \times 144\left(\frac{144+12}{12}\right)S_j^2 + 1$

**Table 7. Transformed series of Deviations of the Observed Values from Means ( $Z_{ij} = \left| y_{ij} - \bar{y}_{,j} \right|$ )**

| | **Jan.** | **Feb.** | **Mar.** | **Apr.** | **May** | **Jun.** | **Jul.** | **Aug.** | **Sept.** | **Oct.** | **Nov.** | **Dec.** | **total** | $\bar{z}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2008 | 0.64 | 1.08 | 0.81 | 0.67 | 0.20 | 0.56 | 1.42 | 0.37 | 0.19 | 0.57 | 0.32 | 0.40 | 7.22 | 0.60 | 0.36 |
| 2009 | 0.64 | 0.16 | 0.17 | 0.02 | 0.36 | 0.00 | 0.03 | 0.37 | 0.11 | 0.00 | 0.30 | 0.17 | 2.35 | 0.20 | 0.19 |
| 2010 | 0.46 | 0.16 | 0.02 | 0.17 | 0.00 | 0.00 | 0.53 | 0.73 | 0.48 | 0.12 | 0.00 | 0.11 | 2.79 | 0.23 | 0.25 |
| 2011 | 0.24 | 0.17 | 0.86 | 0.33 | 0.24 | 0.25 | 0.38 | 0.40 | 0.21 | 0.19 | 0.19 | 0.09 | 3.55 | 0.30 | 0.20 |
| 2012 | 0.49 | 0.31 | 0.12 | 0.48 | 0.00 | 0.62 | 0.72 | 0.22 | 1.32 | 0.53 | 0.46 | 0.47 | 5.74 | 0.48 | 0.34 |
| 2013 | 0.27 | 0.02 | 0.23 | 0.58 | 0.36 | 0.70 | 0.19 | 0.84 | 0.50 | 0.75 | 0.67 | 0.12 | 5.23 | 0.44 | 0.27 |
| 2014 | 0.34 | 0.71 | 0.02 | 0.17 | 0.58 | 0.36 | 0.19 | 0.06 | 0.11 | 0.19 | 0.52 | 0.23 | 3.49 | 0.29 | 0.22 |
| 2015 | 0.56 | 0.71 | 0.23 | 0.24 | 0.26 | 0.33 | 0.19 | 0.22 | 0.09 | 0.06 | 0.23 | 0.16 | 3.31 | 0.28 | 0.19 |
| 2016 | 0.20 | 0.31 | 0.44 | 0.45 | 0.07 | 0.46 | 0.03 | 0.14 | 0.21 | 0.57 | 0.10 | 0.21 | 3.19 | 0.27 | 0.18 |
| 2017 | 0.20 | 0.42 | 0.12 | 0.07 | 0.69 | 0.84 | 0.66 | 0.14 | 0.50 | 1.04 | 0.64 | 0.04 | 5.37 | 0.45 | 0.33 |
| 2018 | 0.34 | 1.08 | 0.02 | 0.17 | 0.44 | 0.33 | 0.32 | 0.55 | 0.35 | 0.53 | 0.54 | 0.09 | 4.75 | 0.40 | 0.28 |
| 2019 | 0.36 | 0.16 | 0.86 | 0.27 | 0.33 | 0.33 | 0.38 | 0.40 | 0.35 | 0.64 | 0.54 | 0.32 | 4.93 | 0.41 | 0.19 |
| Total | **4.74** | **5.30** | **3.90** | **3.63** | **3.53** | **4.80** | **5.04** | **4.43** | **4.42** | **5.19** | **4.53** | **2.40** | 51.91 | | |
| $\bar{z}_{.j}$ | **0.40** | **0.44** | **0.32** | **0.30** | **0.29** | **0.40** | **0.42** | **0.37** | **0.37** | **0.43** | **0.38** | **0.20** | | 0.36 | |
| $\sigma_{.j}$ | **0.16** | **0.37** | **0.34** | **0.20** | **0.21** | **0.25** | **0.38** | **0.24** | **0.34** | **0.32** | **0.22** | **0.13** | | | 0.27 |

**Table 8. Transformed Series of square of deviations of the observed values from seasonal means**

| | $\left(z_{ij}-\bar{z}_{.j}\right)^2$ | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **Jan.** | **Feb.** | **Mar.** | **Apr.** | **May** | **Jun.** | **Jul.** | **Aug.** | **Sept.** | **Oct.** | **Nov.** | **Dec.** | **total** |
| 2008 | 0.06 | 0.41 | 0.24 | 0.14 | 0.01 | 0.02 | 0.99 | 0.00 | 0.03 | 0.02 | 0.00 | 0.04 | 1.96 |
| 2009 | 0.06 | 0.08 | 0.02 | 0.08 | 0.00 | 0.16 | 0.15 | 0.00 | 0.07 | 0.18 | 0.01 | 0.00 | 0.81 |
| 2010 | 0.00 | 0.08 | 0.09 | 0.02 | 0.08 | 0.16 | 0.01 | 0.13 | 0.01 | 0.10 | 0.14 | 0.01 | 0.84 |
| 2011 | 0.02 | 0.07 | 0.29 | 0.00 | 0.00 | 0.02 | 0.00 | 0.00 | 0.02 | 0.06 | 0.03 | 0.01 | 0.54 |
| 2012 | 0.01 | 0.02 | 0.04 | 0.03 | 0.08 | 0.05 | 0.09 | 0.02 | 0.91 | 0.01 | 0.01 | 0.07 | 1.34 |
| 2013 | 0.01 | 0.18 | 0.01 | 0.08 | 0.00 | 0.09 | 0.05 | 0.22 | 0.02 | 0.10 | 0.08 | 0.01 | 0.85 |
| 2014 | 0.00 | 0.07 | 0.09 | 0.02 | 0.08 | 0.00 | 0.05 | 0.10 | 0.07 | 0.06 | 0.02 | 0.00 | 0.57 |
| 2015 | 0.03 | 0.07 | 0.01 | 0.00 | 0.00 | 0.00 | 0.05 | 0.02 | 0.07 | 0.14 | 0.02 | 0.00 | 0.43 |
| 2016 | 0.04 | 0.02 | 0.01 | 0.02 | 0.05 | 0.00 | 0.15 | 0.05 | 0.02 | 0.02 | 0.08 | 0.00 | 0.47 |
| 2017 | 0.04 | 0.00 | 0.04 | 0.05 | 0.16 | 0.20 | 0.06 | 0.05 | 0.02 | 0.37 | 0.07 | 0.03 | 1.08 |
| 2018 | 0.00 | 0.41 | 0.09 | 0.02 | 0.02 | 0.00 | 0.01 | 0.03 | 0.00 | 0.01 | 0.03 | 0.01 | 0.64 |
| 2019 | 0.002 | 0.08 | 0.29 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.04 | 0.03 | 0.01 | 0.46 |
| Total | 0.28 | 1.48 | 1.24 | 0.46 | 0.50 | 0.71 | 1.63 | 0.63 | 1.24 | 1.10 | 0.52 | 0.19 | **9.99** |

**Table 9. Calculation of** $m\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ **m =12**

| $\bar{z}_{.j}$ | $\bar{z}_{..}$ | $\bar{z}_{.j}-\bar{z}_{..}$ | $\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ | $12\times\left(\bar{z}_{.j}-\bar{z}_{..}\right)^2$ |
|---|---|---|---|---|
| 0.4 | 0.36 | 0.040 | 0.002 | 0.019 |
| 0.44 | 0.36 | 0.080 | 0.006 | 0.077 |
| 0.32 | 0.36 | -0.040 | 0.002 | 0.019 |
| 0.3 | 0.36 | -0.060 | 0.004 | 0.043 |
| 0.29 | 0.36 | -0.070 | 0.005 | 0.059 |
| 0.4 | 0.36 | 0.040 | 0.002 | 0.019 |
| 0.42 | 0.36 | 0.060 | 0.004 | 0.043 |
| 0.37 | 0.36 | 0.010 | 0.000 | 0.001 |
| 0.37 | 0.36 | 0.010 | 0.000 | 0.001 |
| 0.43 | 0.36 | 0.070 | 0.005 | 0.059 |
| 0.38 | 0.36 | 0.020 | 0.000 | 0.005 |
| 0.2 | 0.36 | -0.160 | 0.026 | 0.307 |
| | | | | **0.653** |

From Appendix B and Table 9

$$W=\frac{12*(12-1)(0.653)}{(12-1)(9.99)} = \frac{86.196}{109.89} = 0.78$$

# 4 Concluding Remarks

This article has discussed the Chi-Square test in time series data. The Chi-Square test is based on Chi-Square distribution. Although time series data does not satisfy all the assumptions of most common statistical tests, the Chi-Square test appears to be the most efficient among them. The Chi-Square test is able to identify the mixed model successfully in one hundred (100) out of the one hundred (100) simulations. Secondly, the transformed series shown in Appendix B and Table 9 admits additive model. This further confirms that the appropriate model of original data given in Appendix A and Table 5 is multiplicative. In view of this, it is recommended that a study data should be evaluated for assumptions of time series model before applying test for choice of model.

## Competing Interests

Authors have declared that no competing interests exist.

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**Appendix A. Actual series of church marriages in Imo State, Nigeria (2008 – 2019)**

| | Jan. | Feb. | Mar. | Apr. | May | Jun. | Jul. | Aug. | Sept. | Oct. | Nov. | Dec. | total | $\bar{x}_{i.}$ | $\sigma_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2008 | 6 | 2 | 16 | 12 | 17 | 4 | 1 | 15 | 12 | 30 | 26 | 24 | 165 | 13.75 | 9.53 |
| 2009 | 6 | 5 | 6 | 23 | 20 | 7 | 4 | 15 | 11 | 17 | 14 | 30 | 152 | 13.82 | 8.28 |
| 2010 | 18 | 5 | 7 | 28 | 14 | 7 | 7 | 5 | 16 | 15 | 19 | 32 | 166 | 15.09 | 9.02 |
| 2011 | 9 | 7 | 3 | 17 | 11 | 9 | 6 | 7 | 8 | 14 | 23 | 39 | 153 | 12.75 | 9.88 |
| 2012 | 7 | 8 | 8 | 38 | 14 | 13 | 2 | 13 | 37 | 10 | 30 | 57 | 229 | 20.82 | 17.16 |
| 2013 | 15 | 6 | 9 | 42 | 20 | 14 | 5 | 24 | 6 | 36 | 37 | 40 | 245 | 22.27 | 14.37 |
| 2014 | 16 | 12 | 7 | 28 | 25 | 10 | 5 | 11 | 11 | 14 | 32 | 45 | 216 | 18.00 | 11.99 |
| 2015 | 20 | 12 | 9 | 30 | 18 | 5 | 5 | 13 | 9 | 18 | 15 | 42 | 187 | 17.00 | 10.96 |
| 2016 | 14 | 8 | 11 | 15 | 13 | 11 | 4 | 12 | 8 | 30 | 21 | 29 | 176 | 14.67 | 8.09 |
| 2017 | 14 | 9 | 8 | 22 | 7 | 3 | 8 | 9 | 6 | 6 | 10 | 37 | 131 | 11.91 | 9.72 |
| 2018 | 16 | 2 | 7 | 28 | 9 | 5 | 3 | 6 | 7 | 10 | 11 | 39 | 136 | 12.36 | 11.42 |
| 2019 | 8 | 5 | 3 | 18 | 10 | 5 | 6 | 7 | 7 | 32 | 11 | 26 | 138 | 11.50 | 9.12 |
| Total | **149** | **81** | **40** | **301** | **178** | **93** | **56** | **137** | **138** | **232** | **249** | **440** | | | |
| $\bar{y}_{.j}$ | **12.42** | **6.75** | **8.00** | **25.08** | **14.83** | **7.75** | **4.67** | **11.42** | **11.50** | **19.33** | **20.75** | **36.67** | | | |
| $\sigma_{.j}$ | **4.94** | **3.28** | **5.57** | **9.07** | **5.31** | **3.62** | **2.02** | **5.25** | **8.54** | **10.00** | **9.03** | **9.21** | | | |

**Appendix B. Transformed series of church marriages in Imo State, Nigeria (2008 – 2019)**

| | JAN | FEB | MAR | APR | MAY | JUN | JUL | AUG | SEP | OCT | NOV | DEC | $T_{i.}$ | $\bar{X}_{i.}$ | $\hat{\sigma}_{i.}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **2008** | 1.7918 | 0.6931 | 2.7726 | 2.4849 | 2.8332 | 1.3863 | 0.0000 | 2.7081 | 2.4849 | 3.4012 | 3.2581 | 3.1781 | 26.9922 | 2.2494 | 1.0693 |
| **2009** | 1.7918 | 1.6094 | 1.7918 | 3.1355 | 2.9957 | 1.9459 | 1.3863 | 2.7081 | 2.3979 | 2.8332 | 2.6391 | 3.4012 | 28.6358 | 2.3863 | 0.6629 |
| **2010** | 2.8904 | 1.6094 | 1.9459 | 3.3322 | 2.6391 | 1.9459 | 1.9459 | 1.6094 | 2.7726 | 2.7081 | 2.9444 | 3.4657 | 29.8091 | 2.4841 | 0.6478 |
| **2011** | 2.1972 | 1.9459 | 1.0986 | 2.8332 | 2.3979 | 2.1972 | 1.7918 | 1.9459 | 2.0794 | 2.6391 | 3.1355 | 3.6636 | 27.9253 | 2.3271 | 0.6728 |
| **2012** | 1.9459 | 2.0794 | 2.0794 | 3.6376 | 2.6391 | 2.5649 | 0.6931 | 2.5649 | 3.6109 | 2.3026 | 3.4012 | 4.0431 | 31.5622 | 2.6302 | 0.9298 |
| **2013** | 2.7081 | 1.7918 | 2.1972 | 3.7377 | 2.9957 | 2.6391 | 1.6094 | 3.1781 | 1.7918 | 3.5835 | 3.6109 | 3.6889 | 33.5321 | 2.7943 | 0.796 |
| **2014** | 2.7726 | 2.4849 | 1.9459 | 3.3322 | 3.2189 | 2.3026 | 1.6094 | 2.3979 | 2.3979 | 2.6391 | 3.4657 | 3.8067 | 32.3738 | 2.6978 | 0.649 |
| **2015** | 2.9957 | 2.4849 | 2.1972 | 3.4012 | 2.8904 | 1.6094 | 1.6094 | 2.5649 | 2.1972 | 2.8904 | 2.7081 | 3.7377 | 31.2866 | 2.6072 | 0.6448 |
| **2016** | 2.6391 | 2.0794 | 2.3979 | 2.7081 | 2.5649 | 2.3979 | 1.3863 | 2.4849 | 2.0794 | 3.4012 | 3.0445 | 3.3673 | 30.5509 | 2.5459 | 0.5655 |
| **2017** | 2.6391 | 2.1972 | 2.0794 | 3.091 | 1.9459 | 1.0986 | 2.0794 | 2.1972 | 1.7918 | 1.7918 | 2.3026 | 3.6109 | 26.825 | 2.2354 | 0.6476 |
| **2018** | 2.7726 | 0.6931 | 1.9459 | 3.3322 | 2.1972 | 1.6094 | 1.0986 | 1.7918 | 1.9459 | 2.3026 | 2.3979 | 3.6636 | 25.7508 | 2.1459 | 0.8472 |
| **2019** | 2.0794 | 1.6094 | 1.0986 | 2.8904 | 2.3026 | 1.6094 | 1.7918 | 1.9459 | 1.9459 | 3.4657 | 2.3979 | 3.2581 | 26.3952 | 2.1996 | 0.7051 |
| $T_{.j}$ | **29.224** | **21.278** | **23.551** | **37.916** | **31.621** | **23.307** | **17.002** | **28.097** | **27.496** | **33.958** | **35.306** | **42.885** | **351.639** | | |
| $\bar{X}_{.j}$ | **2.4353** | **1.7732** | **1.9625** | **3.1597** | **2.6351** | **1.9422** | **1.4168** | **2.3414** | **2.2913** | **2.8299** | **2.9422** | **3.5737** | | **2.4419** | |
| $\hat{\sigma}_{.j}$ | **0.4428** | **0.5896** | **0.4773** | **0.3758** | **0.3746** | **0.4889** | **0.5835** | **0.4537** | **0.5108** | **0.5519** | **0.45** | **0.2472** | | | 0.7493 |



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